Student Number: ………………………..

Barker College

2012

YEAR 12
TRIAL HSC
EXAMINATION

MATHEMATICS

Staff Involved:

- **GIC** • **JMW**
- **BJB** • **DZP**
- **PJR** • **ASW**
- **KJL*** • **RMH**
- **LMD***

TUESDAY 31st JULY 2012

130 copies

Multiple Choice Answer Sheet

Choose the best response and fill in the response oval completely

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Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

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General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Write your Barker Student Number on all pages of your answers
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper which may be detached for your use
- Show ALL necessary working in Questions 11-16
- Marks may be deducted for careless or badly arranged working

Total marks - 100

SECTION I Pages 3 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

SECTION II Pages 7 – 16

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section.

Section I – **Multiple Choice**
Attempt Questions 1 - 10
All questions are of equal value

Answer each question on the multiple choice answer sheet.

---

1. The value of $\log_6 2000$ is closest to:

   (A) 1.3

   (B) 4.2

   (C) 5.8

   (D) 9.4

2. A circle has the equation $x^2 - 6x + y^2 - 1 = 0$. It has a radius of:

   (A) 10

   (B) 3

   (C) 1

   (D) $\sqrt{10}$

3. If $3\sqrt{2} + \sqrt{8} = \sqrt{a}$, the value of a is:

   (A) 98

   (B) 90

   (C) 50

   (D) 48

Section I – Multiple Choice continued

4. A function $y = f(x)$ has $f'(3) = 0$ and $f''(3) = -1$. At the point where $x = 3$, $y = f(x)$ is:

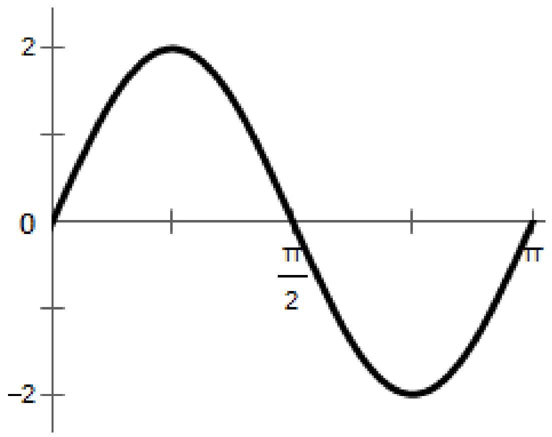
(A) Decreasing and concave up

(B) Stationary and concave down

(C) Stationary and concave up

(D) Stationary with a horizontal point of inflexion

5. The equation of the trigonometric function below could be:



(A) $f(x) = 2\sin(2x)$

(B) $f(x) = \frac{1}{2}\sin(2x)$

(C) $f(x) = \frac{1}{2}\sin\left(\frac{x}{2}\right)$

(D) $f(x) = 2\sin\left(\frac{x}{2}\right)$

Section I – Multiple Choice continued

6. The range of the function $y = \frac{(\ln x)^2}{-3}$ is given by:

(A) $y \geq 0$

(B) $y \leq 0$

(C) All real y values

(D) All real y values, $y \neq 0$

7. The value of $\sum_{n=3}^{6} \frac{3-n}{2}$ is:

(A) -3

(B) $\frac{-3}{2}$

(C) 0

(D) 3

8. An expression which is **not** equal to $2^{3x} \times 3^{3x}$ is:

(A) 6^{3x}

(B) $2^{3x} \times 9^{\frac{3x}{2}}$

(C) $\frac{2^{3x} \times 3^{3x-1}}{3}$

(D) $\frac{2^{3x}}{3^{-3x}}$

Section I – Multiple Choice continued

9. The limiting sum of the series given by 81, 27, 9, 3… is

(A) $\frac{243}{2}$

(B) $-\frac{81}{2}$

(C) 120

(D) $\frac{81}{2}$

10. Given the arithmetic sequence $1,\ 2+x,\ \ldots,$ the tenth term of the sequence will be

(A) $(2+x)^9$

(B) $(2+x)^{10}$

(C) $10+x$

(D) $10+9x$

End of Section I

Section II – Extended Response

Attempt Questions 11 - 16
All questions are of equal value

Answer each question in a separate writing booklet. Extra writing booklets are available.

Marks

Question 11 (15 marks) **[USE A NEW WRITING BOOKLET]**

(a) Fully factorise $x^3 - 8$ **2**

(b) Solve $2\sin x = \sqrt{3}$, for $0 \le x \le 2\pi$ **2**

(c) Differentiate with respect to *x*.

(i) $2x^3 + \sqrt{x}$ **2**

(ii) $\dfrac{-\cos 2x}{x}$ **2**

(d) Solve: $x^2 - 6 > x$ **2**

(e) Sketch $y = |2x - 4|$ including any intercepts. **2**

(f) Evaluate $\displaystyle\int_1^2 \left(4t - \frac{3}{t^2}\right) dt$ **3**

End of Question 11

Marks

Question 12 (15 marks) **[USE A NEW WRITING BOOKLET]**

(a) Consider the parabola $(y+7)^2 = 12(x-3)$.

(i) Write down the coordinates of the vertex. **1**

(ii) Write down the equation of the directrix. **1**

(iii) Sketch the parabola, showing these features. **1**

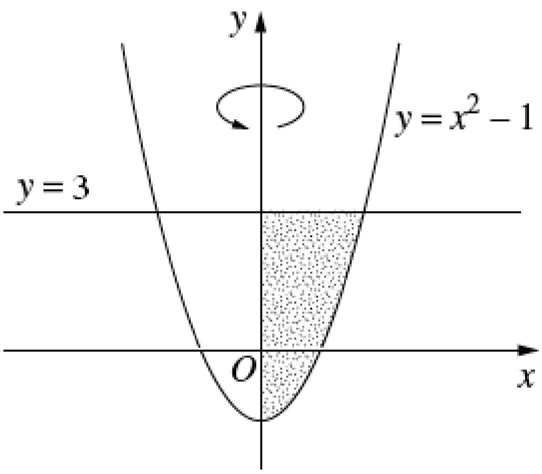
(b) Find

(i) $\displaystyle\int \frac{-x}{2+3x^2}\,dx$ **2**

(ii) $\displaystyle\int \frac{1}{\sqrt{1+2x}}\,dx$ **2**

(c) Prove that $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$ **2**

(d) In the diagram, the shaded region is bounded by the parabola $y = x^2 - 1$, the y-axis and the line $y = 3$. Find the volume of the solid formed when the shaded region is rotated about the y-axis. **3**



Question 12 continues on page 10

Marks

Question 12 continued

(e) A straight line has the equation $y = x + k$.

The perpendicular distance from the point (2, 7) to this straight line is $\frac{1}{\sqrt{2}}$ units.

Find all possible values of k. **3**

End of Question 12

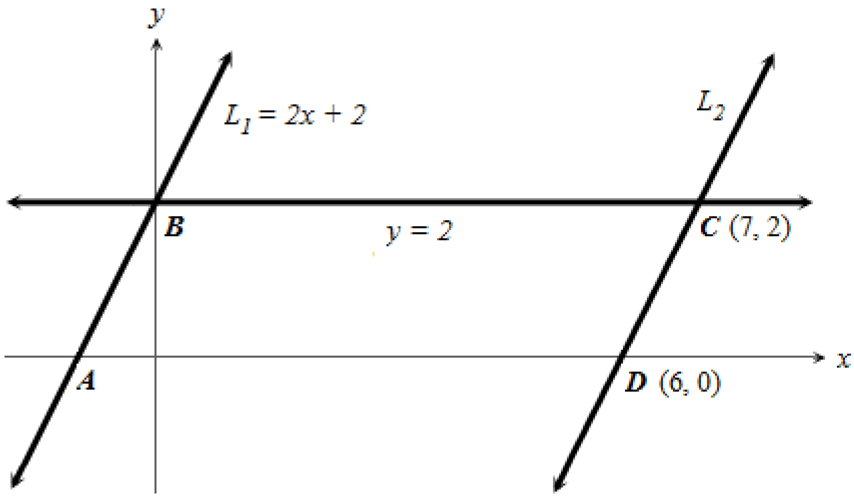
Marks

Question 13 (15 marks) **[USE A NEW WRITING BOOKLET]**

(a) In the diagram, two lines L_1 and L_2 intersect with the line $y = 2$.

L_1 has equation $y = 2x + 2$ and meets the x- and y-axes at A and B respectively.

L_2 meets $y = 2$ at the point C (7, 2) and has an intercept at D (6, 0).



(i) Find the coordinates of A. **1**

(ii) Find the equation of L_2. **2**

(iii) Find the area of the parallelogram ABCD. **1**

(b) Find the equation of the normal to $f(x) = 2\tan x$, at the point where $x = \dfrac{\pi}{4}$ **3**

(c) Use Simpson's rule with five function values to approximate $\int_{-2}^{2} \sqrt{16 - x^2}\, dx$.

Give your answer to 3 significant figures. **3**

Question 13 continues on page 12

Marks

Question 13 continued

(d) Let $f(x) = x - \ln(x+1)$

(i) Determine the domain of this function. **1**

(ii) Find the coordinates of any stationary points and determine their nature. **3**

(iii) Hence sketch $f(x) = x - \ln(x+1)$, including any intercepts. **1**

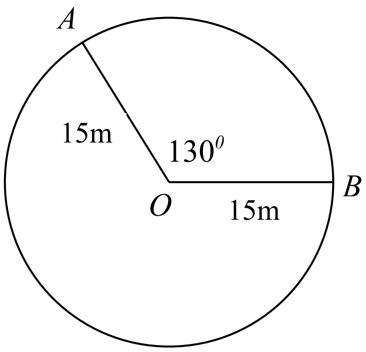
End of Question 13

Marks

Question 14 (15 marks) **[USE A NEW WRITING BOOKLET]**

(a) Find the exact value of $\cot\left(\frac{2\pi}{3}\right) - \cos\left(\frac{7\pi}{3}\right)$. **2**

(b) Two radii are drawn on a circle of radius 15m, meeting the circle at A and B. They form an angle of 130 degrees at the centre.



NOT TO SCALE

(i) Find the length of the minor arc AB. Leave your answer in exact form. **2**

(ii) Find the area of the minor sector AOB. Leave your answer in exact form. **1**

(c) The quadratic equation $x^2 + mx + k = 0$ has one root that is twice the value of the other.

Prove that $2m^2 = 9k$. **3**

(d) Explain why the function $f(x) = x^3 + 5x$ has no stationary points. **2**

Question 14 continues on page 14

Marks

Question 14 continued

(e) The hose from a fire truck is gradually turned off, so that the flow rate of water (V litres) from the hose at time (t seconds) is given by $\dfrac{dV}{dt} = \dfrac{t}{2} - 3$.

(i) At what time was the hose fully turned off? **1**

(ii) Find V as a function of t if the truck contained 145L of water after 2 seconds. **2**

(iii) Hence or otherwise, find the total volume of water that is released during the time the hose is turned off. **2**

End of Question 14

Marks

Question 15 (15 marks) **[USE A NEW WRITING BOOKLET]**

(a) Mr Rundle can see Ms Blowes and Mrs Clarke in the distance. He calculates that Ms Blowes is at a bearing of 030^0 and Mrs Clarke is at a bearing of 314^0.

They are at a distance of 32m and 42m from Mr Rundle respectively.

What is the distance between Ms Blowes and Mrs Clarke?
Give your answer to the nearest metre. **2**

(b) Find the sum of the first 10 terms of this series: $\sqrt{5},\ 5,\ 5\sqrt{5},\ 25...$
Leave your answer in exact form. **2**

(c) Over a number of years, Mr Peattie's Rugby Club sells season ticket subscriptions that can be approximated by the exponential growth model,

$$N = Ae^{kt}$$

where N is the number of subscribers and t is the time in years after 1 January 2012.

(i) It is estimated that at the start of 2012, when $t = 0$, there will be 500 subscribers, and that at the start of 2014, there will be 1200 subscribers. Find A and k, leaving your answers in exact form. **3**

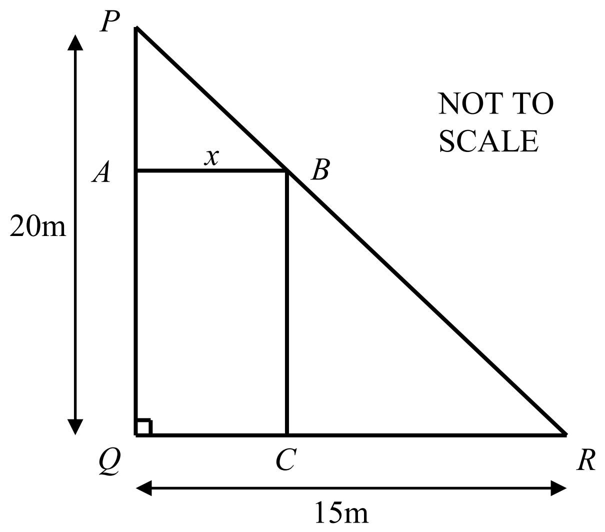
(ii) According to the model, during which year will the number of subscribers exceed 10 000? **2**

Question 15 continues on page 16

Marks

Question 15 continued

(d) In the right-angled ΔPQR, $PQ = 20$m and $QR = 15$m.
$QABC$ is a rectangle inscribed in ΔPQR, with $AB = x$.



(i) Prove that $\Delta PAB \parallel\!| \Delta PQR$. **2**

(ii) Hence or otherwise, show that $AP = \dfrac{4x}{3}$ **1**

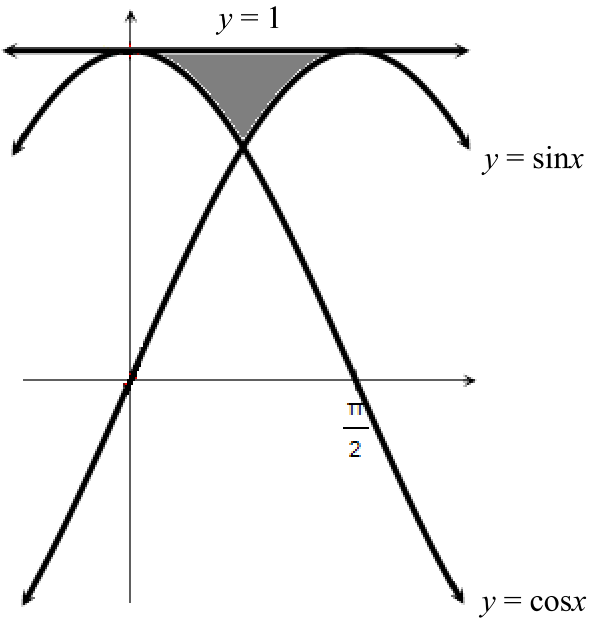
(iii) As B moves, the area of rectangle $QABC$ changes.
Find the maximum possible area of the rectangle $QABC$. **3**

End of Question 15

Marks

Question 16 (15 marks) **[USE A NEW WRITING BOOKLET]**

(a) The shaded region in the diagram is bounded by the curves $y = \sin x$, $y = \cos x$, and $y = 1$.



(i) Find the coordinates of the point of intersection of $y = \sin x$ and $y = \cos x$. **1**

(ii) Hence or otherwise, find the area of the shaded region. **3**

Question 15 continues on page 18

Marks

Question 16 continued

(b) The line $y = mx$ is a tangent to the curve $y = e^{2x}$.

(i) Draw a sketch of $y = mx$ and $y = e^{2x}$ on the same set of axes, showing this information. **1**

(ii) Find the value of m. **3**

(c) Ashley has just retired with \$1 000 000 in her bank account. This account attracts interest at a rate of 5% per annum compounded annually.

She intends to withdraw money in equal amounts at the end of each year, and wants the money in her account to last for exactly 20 years.

Let A_n be the amount of money in her account after n years.

(i) Write down an expression for the amount left after two withdrawals. **1**

(ii) Calculate the amount of each withdrawal. **3**

(iii) After her tenth withdrawal, the bank changes its interest rate to 3% p.a. on the balance remaining in her account. Interest is still compounded annually.

Ashley changes the amount of her withdrawals so that they last the remaining 10 years and are equal in value over these 10 years.

What will be the new amount of each withdrawal in these last 10 years? **3**

End of Question 16
End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \, ; \quad x \neq 0, \ \text{ if } \ n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Year 12 Mathematics
Trial HSC 2012
Solutions

1. $\log_6 2000 = \frac{\ln 2000}{\ln 6}$

OR $\frac{\log 2000}{\log 6} \approx 4.242...$ (B)

2. $x^2 - 6x + y^2 - 1 = 0$

$x^2 - 6x + 9 + y^2 = 1 + 9$

$(x-3)^2 + y^2 = 10$

circle centre $(3,0)$

radius $= \sqrt{10}$ (D)

3. $3\sqrt{2} + \sqrt{8}$

$= 3\sqrt{2} + 2\sqrt{2}$

$= 5\sqrt{2} = \sqrt{50}$ (C)

4. When $x = 3$,

gradient 0

concave down (B)

5. Amplitude = 2

Period $= \pi$ (A)

6. $y = \frac{(\ln x)^2}{-3}$

numerator always +ve

denominator always −ve

$\therefore y \leq 0$ (B)

7. $\sum_{n=3}^{6} \frac{3-n}{2} = 0 - \frac{1}{2} - 1 - \frac{3}{2}$

$= -3$ (A)

8. $2^{3x} \times 3^{3x}$

$= (2^3)^x \times (3^3)^x$

$= 8^x \times 27^x$ (C)

9. $S_\infty = \frac{a}{1-r} = \frac{81}{1-\frac{1}{3}}$

$= \frac{243}{2}$ (A)

10. $a = 1$ $d = 2 + x - 1 = 1 + x$

$T_{10} = a + (n-1)d$

$= 1 + 9(1+x)$

$= 10 + 9x$ (D)

11a) $x^3 - 8 = (x-2)(x^2+2x+4$

b) $2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$

c) (i) $y = 2x^3 + \sqrt{x}$

$= 2x^3 + x^{\frac{1}{2}}$

$y' = 6x^2 + \frac{1}{2}x^{-\frac{1}{2}}$

(ii) $y = -\frac{\cos 2x}{x}$

$y' = \frac{x(\sin 2x)(2) + (\cos 2x)(1}{x^2}$

$= \frac{2x\sin 2x + \cos 2x}{x^2}$

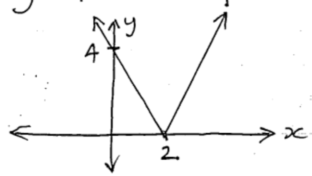
d) $x^2 - 6 > x$

$x^2 - x - 6 > 0$

$(x-3)(x+2) > 0$

$x < -2$ or $x > 3$

e) $y = |2x - 4|$



11f) $\int_1^2 4t - \frac{3}{t^2}\, dt$

$= \left[\frac{4t^2}{2} - \frac{3t^{-1}}{-1}\right]_1^2$

$= \left[2t^2 + 3t^{-1}\right]_1^2$

$= \left(2\times 2^2 + \frac{3}{2}\right) - (2+3)$

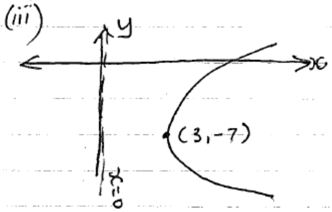
$= \left(8 + \frac{3}{2}\right) - 5 = \frac{9}{2}$

12a) $(y+7)^2 = 12(x-3)$

(i) vertex $(3,-7)$

(ii) $a = 3$ $\therefore x = 0$

(iii)



b)(i) $\int \frac{-x}{2+3x^2}\, dx$

$= -\frac{1}{6}\int \frac{6x}{2+3x^2}\, dx$

$= -\frac{1}{6}\ln(2+3x^2) + C$

(ii) $\int \frac{1}{\sqrt{1+2x}}\, dx$

$= \int (1+2x)^{-\frac{1}{2}}\, dx$

$= \frac{(1+2x)^{\frac{1}{2}}}{\frac{1}{2}\times 2} + c$

$= \sqrt{1+2x} + c$

c) LHS $= \frac{1-\sin\theta}{1-\sin^2\theta} + \frac{1+\sin\theta}{1-\sin^2\theta}$

$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$

$= 2\sec^2\theta$ = RHS

d) $\int_{-1}^{3} \pi x^2\, dy = \int_{-1}^{3} \pi(y+1)\, dy$

$V = \pi\left[\frac{y^2}{2} + y\right]_{-1}^{3}$

$= \pi\left[\left(\frac{9}{2}+3\right) - \left(\frac{1}{2}-1\right)\right]$

$= \pi\left[7\frac{1}{2} + \frac{1}{2}\right] = 8\pi\ u^3$

e) $y = x + k$

$x - y + k = 0$ $(2,7)$

$\perp$ dist $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$\frac{1}{\sqrt{2}} = \frac{|1\times 2 - 1\times 7 + k|}{\sqrt{1^2+1^2}}$

$\frac{1}{\sqrt{2}} = \frac{|2 - 7 + k|}{\sqrt{2}}$

$\therefore 1 = |-5 + k|$

$k = 4$ or $k = 6$

13a) (i) $A(-1, 0)$

(ii) $m = 2$ $y - 0 = 2(x-6)$

$y = 2x - 12$

$2x - y - 12 = 0$

(iii) $A = bh$

$= 7\times 2 = 14\ u^2$

13b) $f(x) = 2\tan x$

$f'(x) = 2\sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2\sec^2\left(\frac{\pi}{4}\right)$

$= \dfrac{2}{\cos^2\left(\frac{\pi}{4}\right)}$

$= \dfrac{2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 4$

$\therefore$ grad of normal $= -\frac{1}{4}$

point $\left(\frac{\pi}{4}, 2\right)$

$y - 2 = -\frac{1}{4}\left(x - \frac{\pi}{4}\right)$

$y = -\frac{1}{4}x + 2 + \frac{\pi}{16}$

$16y = -4x + 32 + \pi$

$4x + 16y - 32 - \pi = 0$

13c) $\int_{-2}^{2} \sqrt{16 - x^2}\, dx$

x	-2	-1	0	1	2
y	$\sqrt{12}$	$\sqrt{15}$	4	$\sqrt{15}$	$\sqrt{12}$
weight	1	4	2	4	1

$h = 1$

$A \approx \frac{1}{3}\left(\sqrt{12} + 4\sqrt{15} + 4\times 2 + 4\sqrt{15} + \sqrt{12}\right)$

≈ 15.30402333

≈ 15.3 (3 sig figs) u^2

d) $f(x) = x - \ln(x+1)$

(i) Domain $x > -1$

(ii) $f'(x) = 1 - \dfrac{1}{x+1}$

$f'(x) = 0$ at stat points.

$1 - \dfrac{1}{x+1} = 0$

$1 = \dfrac{1}{x+1}$

$x + 1 = 1$

$x = 0, \; y = 0$

$\therefore$ stat point at $(0, 0)$

x	-1	$-\frac{1}{2}$	0	1
$\frac{dy}{dx}$	undef	-1	0	$\frac{1}{2}$
		\	—	/

minimum $(0, 0)$

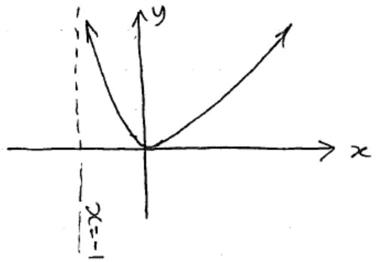
OR $f''(x) = --(x+1)^{-2}$

$= (x+1)^{-2}$

when $x = 0$ $f''(0) = 1$

$\therefore$ minimum

(iii)



14 a) $\cot\left(\frac{2\pi}{3}\right) - \cos\left(\frac{7\pi}{3}\right)$

$= \dfrac{1}{\tan\left(\frac{2\pi}{3}\right)} - \cos\frac{\pi}{3}$

$= -\dfrac{1}{\sqrt{3}} - \dfrac{1}{2}$

$= \dfrac{-2\sqrt{3} - 3}{6} = \dfrac{-\sqrt{3}}{3} - \dfrac{1}{2}$

b) (i) $l = r\theta \qquad \theta = \frac{130}{180}\pi$

$= 15 \times \frac{13\pi}{18} \qquad = \frac{13}{18}\pi$

$= \dfrac{65\pi}{6}$ m

(ii) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 15^2 \times \frac{13}{18}\pi$

$= \dfrac{325}{4}\pi$ m^2

c) $x^2 + mx + k = 0$

$\beta = 2\alpha$

$(x - \alpha)(x - 2\alpha) = x^2 + mx + k$

$x^2 - 2x\alpha - \alpha x + 2\alpha^2 = x^2 + mx + k$

$-3\alpha x + 2\alpha^2 = mx + k$

$\therefore m = -3\alpha$

$k = 2\alpha^2$

$2m^2 = 2(-3\alpha)^2 = 18\alpha^2$

$9k = 9 \times 2\alpha^2 = 18\alpha^2$

$\therefore 2m^2 = 9k$

OR

$\alpha + 2\alpha = \frac{-m}{1}$

$3\alpha = -m$

$\alpha \times 2\alpha = k$

$2\alpha^2 = k$ etc

d) $f(x) = x^3 + 5x$

$f'(x) = 3x^2 + 5$

stat points when $f'(x) = 0$

$0 = 3x^2 + 5$

$-5 = 3x^2$ but $3x^2$ is always +ve

$\therefore$ No stationary points.

e) $\dfrac{dV}{dt} = \dfrac{t}{2} - 3$

(i) turned off when $\frac{dV}{dt} = 0$

$0 = \frac{t}{2} - 3$

$t = 6$ seconds.

(ii) $V = \dfrac{t^2}{4} - 3t + c$

when $t = 2$ $V = 145$

$145 = 1 - 6 + c$

$150 = c$

$\therefore V = \dfrac{t^2}{4} - 3t + 150$

(iii) when $t = 0$ $V = 150$ L

when $t = 6$ $V = \frac{36}{4} - 18 + 150$

$= 141$ L

$\therefore$ 9L of water released

OR

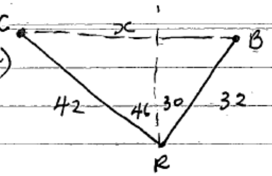
$\int_0^6 \frac{t}{2} - 3\, dt$

$= \left[\frac{t^2}{4} - 3t\right]_0^6$

$= \frac{36}{4} - 18 = -9$

$\therefore$ 9 L of water released.

15a)



$x^2 = 42^2 + 32^2 - 2 \times 45 \times 32 \cos 76$

$x^2 = 2137 \cdot 7139\ldots$

$x = 46 \cdot 235\ldots$

$x = 46\text{m}$ (nearest m)

b) $a = \sqrt{5}$ $r = \sqrt{5}$

$S_{10} = \dfrac{\sqrt{5}\left((\sqrt{5})^{10} - 1\right)}{\sqrt{5} - 1}$

$= \dfrac{\sqrt{5}\,(3125 - 1)}{\sqrt{5} - 1}$

c) $N = Ae^{kt}$

$\dfrac{dN}{dt} = kAe^{kt} = kN$

(i) when $t = 0$ $N = 500$

$A = 500$

$\therefore N = 500e^{kt}$

when $t = 2$ $N = 1200$

$1200 = 500e^{2k}$

$\dfrac{12}{5} = e^{2k}$

$k = \frac{1}{2} \ln\left(\frac{12}{5}\right)$

(ii) $N > 10000$

Find the equality: $10000 = 500e^{\frac{1}{2}\ln 2\cdot 4 \times t}$

$20 = e^{\frac{1}{2}\ln 2\cdot 4t}$

$\ln 20 = \frac{1}{2}\ln 2\cdot 4 \times t$

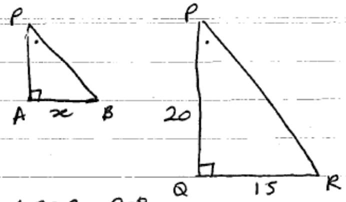
$t = \dfrac{\ln 20}{\frac{1}{2}\ln 2\cdot 4}$

$= 6\cdot 843\ldots$

$\therefore$ During 7th year

$\therefore$ during 2018

d)



(i) $\angle PAB = 90°$ (AB ∥ QC opposite sides of rectangle QABC corresponding angles)

$\angle P$ is common

$\therefore \Delta PAB \parallel\!| \Delta PQR$ (equiangular)

(ii) $\dfrac{AP}{x} = \dfrac{20}{15}$ (ratios of sides on similar triangles)

$\therefore AP = \dfrac{4x}{3}$

(iii) $A = \left(20 - \frac{4x}{3}\right)x$

$= 20x - \dfrac{4x^2}{3}$

$A' = 20 - \dfrac{8x}{3}$ $\qquad A'' = -\dfrac{8}{3}$ $\therefore$ maximum

OR

When is $A' = 0$? For maximum area

$0 = 20 - \dfrac{8x}{3}$

$20 = \dfrac{8x}{3}$

$60 = 8x$

$x = 7\cdot 5$

x	7	7·5	8
A'	$1\frac{1}{3}$	0	$-1\frac{1}{3}$
	/	—	\

maximum

$\therefore$ Max Area $= 7\cdot 5\left(20 - \dfrac{4 \times 7\cdot 5}{3}\right)$

$= 75$ units^2

16a) $y = \sin x$, $y = \cos x$

$\dfrac{\sin x}{\cos x} = \dfrac{\cos x}{\cos x}$

$\tan x = 1$

$x = \dfrac{\pi}{4}$

(i) $\left(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}}\right)$

16a) (ii) Area is symmetrical

$\therefore \frac{1}{2}A = \int_0^{\frac{\pi}{4}} 1 - \cos x \, dx$

$= \left[x - \sin x\right]_0^{\frac{\pi}{4}}$

$= \left(\frac{\pi}{4} - \sin\frac{\pi}{4}\right) - (0 - \sin 0)$

$= \dfrac{\pi}{4} - \dfrac{1}{\sqrt{2}} - 0$

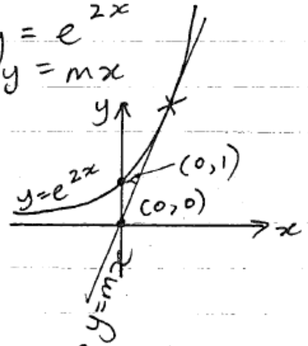
$A = 2\left(\dfrac{\pi}{4} - \dfrac{1}{\sqrt{2}}\right)$

$= \dfrac{\pi}{2} - \dfrac{2}{\sqrt{2}}$ units^2

b) $y = e^{2x}$

$y = mx$

(i)



(ii) $e^{2x} = mx$

$\ln(e^{2x}) = \ln mx$

$2x = \ln mx$ ... ①

$y' = 2e^{2x}$

$m = 2e^{2x}$ .... ②

$2x = \ln 2xe^{2x}$ ② into ①

$e^{2x} = 2xe^{2x}$

$1 = 2x$

$x = \dfrac{1}{2}$

$\therefore m = 2e$

* There are other methods *

c)(i) \$1000000

5% p.a. compounded annually

20 years

at end no money left

(i) $A_1 = 1000000 \times 1.05 - M$

$A_2 = (1000000 \times 1.05 - M) \times 1.05 - M$

$= 1000000 \times 1.05^2 - 1.05M - M$

(ii) $A_3 = A_2 \times 1.05 - M$

$= (1000000 \times 1.05^2 - 1.05M - M)1.05 - M$

$= 1000000 \times 1.05^3 - 1.05^2 M - 1.05M - M$

$A_{20} = 1000000 \times 1.05^{20} - 1.05^{19}M - 1.05^{18}M \ldots - M$

$= 1000000 \times 1.05^{20} - M(1 + 1.05 + 1.05^2 + \ldots + 1.05^{19})$

$= 1000000 \times 1.05^{20} - M\left(\frac{1(1.05^{20} - 1)}{0.05}\right)$

After 20 years $A_{20} = 0$

$0 = 1000000 \times 1.05^{20} - M\left(\frac{1.05^{20} - 1}{0.05}\right)$

$M = \frac{1000000 \times 1.05^{20}}{\left(\frac{1.05^{20} - 1}{0.05}\right)}$

$= \$80242.58719$

(iii) $A_{10} = 1000000 \times 1.05^{10} - 80242.59\left(\frac{1.05^{10} - 1}{0.05}\right)$

$= \$619611.95$

$A_1 = 619611.95 \times 1.03 - M$

$A_2 = A_1 \times 1.03 - M$

$= (619611.95 \times 1.03 - M) \times 1.03 - M$

$A_2 = 619611.95 \times 1.03^2 - M \times 1.03 - M$

$A_3 = 619611.95 \times 1.03^3 - 1.03^2 M - 1.03M -$

$A_{10} = 619611.95 \times 1.03^{10} - M(1 + 1.03 + \ldots 1.03^9)$

$= 619611.95 \times 1.03^{10} - M\left(\frac{1(1.03)^{10} - 1}{0.03}\right)$

But $A_{10} = 0$

$0 = 619611.95 \times 1.03^{10} - M\left(\frac{1.03^{10} - 1}{0.03}\right)$

$M = \frac{619611.95 \times 1.03^{10}}{\left(\frac{1.03^{10} - 1}{0.03}\right)}$

$= \$72637.42$